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# The Chief Accountant and Mathematical Friend of Ramanujan—S. Narayana Aiyar

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**Abstract.** S. Narayana Aiyar was chief accountant at the Madras Port Trust office, where Ramanujan worked as a clerk in 1912–1914. In this article, a short biography of S. Narayana Aiyar is given, his mathematical contributions are discussed, and his personal and mathematical relationships with Ramanujan are examined and emphasized.

**1. INTRODUCING S. NARYANA AIYAR.** In June 2009, the author had the privilege of participating in a conference commemorating K. Venkatachaliengar on the centenary of his birth. For many years, more than any other Indian mathematician, Venkatachaliengar had kept the spirit of Ramanujan’s mathematics alive in his homeland, and so it was fitting that a conference in his memory be held on the beautiful campus of Infosys in Bangalore. At this five-day meeting, a film entitled “God, Zero and Infinity” about the life of Ramanujan was screened for the delegates. At one particular juncture in the film, I was saddened to hear one of the narrators condescendingly proclaim that S. Narayana Aiyar, chief accountant at the Madras Port Trust office, was an “amateur mathematician,” who was unable to appreciate the work of his employee, S. Ramanujan. Since S. Narayana Aiyar was not able to defend himself from this uncharitable remark, this listener decided to construct a defense for him.

Readers familiar with the life of Ramanujan might recall that after receiving a monthly stipend from R. Ramachandra Rao for more than one year, in February, 1912, Ramanujan wrote the Madras Port Trust office seeking employment as a clerk. Having obtained a position at the Port Trust office, Ramanujan was supervised by S. Narayana Aiyar. More than any other person in India during the years 1910–1914 when Ramanujan lived in Madras prior to his departure for England in March, 1914, S. Narayana Aiyar was able to appreciate Ramanujan’s work. It has been related that during the evenings and into the wee hours of the mornings, S. Narayana Aiyar and Ramanujan worked together on mathematics. Of course, we are not claiming that S. Narayana Aiyar was anywhere near the equal of Ramanujan in mathematical ability, but clearly Ramanujan had respect for him, both as chief accountant at the Madras Port Trust office and as a mathematician. Since many readers will have some knowledge and interest in Ramanujan and his work, but will have little familiarity with S. Narayana Aiyar and his influence on Ramanujan, in this brief article we provide a few details about the life of S. Narayana Aiyar. Some of our narration can be found in the author’s article in [9, pp. 97–98]. However, in contrast to our account in [9], we also discuss his mathematical contributions.

Indian names when translated into English often bear different spellings. Another rendition of Aiyar is Iyer, which is the preferred family spelling. Since Aiyar was used in Narayana Aiyar’s mathematical publications, we adhere to this spelling.

**2. CULTURAL BACKGROUND.** We first discuss the religious and cultural background in which Ramanujan and Narayana Aiyar lived to help us understand the relationship between them. Ramanujan and Narayana Aiyar were Tamils, and in Tamil-

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speaking areas of South India, in particular, in the southern Indian state of Tamil Nadu (where Madras, now Chennai, is located), the two primary Hindu Brahmin groups are the Siva-worshipping Iyers (also called Aiyars) and the Vishnu-worshipping Iyengars. Thus, although both were Brahmins, Ramanujan was an Iyengar, and Narayana Aiyar was an Iyer. Orthodox Iyengars are generally more conservative than orthodox Iyers, but perhaps more so at that time. Narayana is another name for the god Vishnu, but the Iyers also use that name! For generations, this community were priests, who were involved mainly in the study of Hindu Sanskrit scriptures and the Vedas, while observing a strict personal and community code. The Vedas (meaning knowledge or wisdom) are the oldest Hindu scriptures. Nonmaterialistic, living simply, and fully backed by the early Hindu rulers, these two Brahmin conclaves were in the top hierarchy of the Hindu caste system, which was a gradation based on professions. The British found it convenient to tap this readily available resource for the administration of the vast Indian continent. Consequently, the Brahmin community gradually began to deviate from their original religious and social pursuits, and so from the mid-1800s to the mid-1900s, there was an explosion from the community of legal luminaries, administrators, and mathematicians that included the prestigious Indian Civil Service, social reformers, and, unfortunately for the British, the Indian freedom fighters. This is a brief history of the background and the social climate in which Narayana Aiyar and Ramanujan were born and lived.

**3. LIFE STORY OF S. NARAYANA AIYAR.** S. Narayana Aiyar was born on December 15, 1874 in Cumbum, near Madurai in Tamil Nadu. (The initial S. stands for his father's name Subbanarayanan.) His parents, Subbanarayanan Iyer and Lakshmi, were poor. Both his father and paternal grandfather, Ananthanarayana Iyer, were Hindu priests, well versed in Sanskrit and the Vedas, and made their livings conducting Hindu rituals in private homes. Narayana Aiyar's family was so poor that once during his early school days, at the annual school inspection, he had to share his only shirt with his elder brother. Having struggled in his early life, he held a lifelong affectionate concern for the welfare of the poor and everyone with whom he worked. He had one elder brother and three sisters.

Narayana Aiyar earned an M.A. in mathematics from St. Joseph's College in Trichinopoly, one of the premier colleges in southern India at that time. In those days, the M.A. degree was the highest degree that was offered in any subject in the absence of the Ph.D., which was introduced only many years later. While teaching at St. Joseph's College, Narayana Aiyar became known to Francis Spring, who at that time was serving with the railways in Trichinopoly. In 1900, Narayana Aiyar was asked to become office manager at the Madras Port Trust office, at the invitation of Francis Spring, who by that time had become chairman of the Madras Port Trust, who was to be knighted in 1911, and who was to become one of Ramanujan's most ardent supporters. In due course, Narayana Aiyar was promoted to chief accountant, his position at the time of Ramanujan's application for employment as a clerk in February, 1912, and thus became the highest-ranking Indian at the Port Trust, where he served until his retirement in 1934. In recognition of his service, the government awarded him the title of "Rao Bahadur," a title of honor issued by the British to individuals who had performed great service to the nation.

Narayana Aiyar and his wife Avibakthanayaki had two daughters, Meenakshi and Kamakshi, and one son, Subbanarayanan. In 1925, Narayana Aiyar's son-in-law (husband of Kamakshi), M. S. Venkataraman (1902–1967), was appointed to fill a vacancy in the Madras Port Trust office. At the time of Indian independence in 1947, he was



S. Narayana Aiyar (1874–1937)

traffic manager and then became the first Indian to officiate as chairman of the Madras Port Trust. Upon assuming the post, he gathered together all of the files connected with Ramanujan and transferred them to the National Archives in New Delhi. Copies of these papers were secured for the present author by Nobel laureate S. Chandrasekhar, and many are reproduced in the author's book *Ramanujan: Letters and Commentary*, coauthored with R. A. Rankin [7]. Narayana Aiyar's son, Subbanarayanan, also joined the Madras Port Trust, serving as assistant section master and retiring as office manager.

One incident in Narayana Aiyar's life is often related with amusement by his descendants. Narayana Aiyar's wife had arranged a prospective alliance for her younger sister and requested that Narayana Aiyar visit the young man and give his valued opinion. Accordingly, Narayana Aiyar called on the prospective bridegroom and asked him but one question. Satisfied with the reply, he gave his approval for the marriage. The question he asked was, "What is the value of  $\pi$ ?"

Narayana Aiyar was known for his simplicity bordering on austerity. Although he could have afforded a more comfortable mode of transportation, he traveled to and from work by tramcar, the ordinary transport for common people. He did not even own a radio, although in the evenings he often visited his daughter Kamakshi to listen to the English news on her radio. He insisted on absolute silence while taking food, as prescribed in the scriptures. He was religious and scrupulously honest, had an independent mind, and was a (silent) champion of Indian independence. Although he suffered from a mild case of diabetes, he took good care of his health and lived a normal life until shortly before he died on January 17, 1937.

As indicated above, after working at the Madras Port Trust office during the day, Narayana Aiyar and Ramanujan often worked on mathematics together at night. It is interesting that Narayana Aiyar communicated to the *Journal of the Indian Mathematical Society* two brief papers summarizing some of Ramanujan's discoveries in the theory of prime numbers, and some applications of his "master theorem" for evaluating integrals [3, 4]. By the time these two papers were communicated, Ramanujan had already published three papers and several problems in the *Journal of the Indian Mathematical Society*, and so one might naturally ask why Ramanujan did not communicate these results himself to the *Journal of the Indian Mathematical Society*. The answer

to this question can be implicitly discerned from the initial correspondence between Ramanujan and Hardy. Hardy had eagerly requested some of Ramanujan's proofs, especially in the theory of primes. However, Ramanujan was very reluctant to provide such proofs. We think that the reason for this was that Ramanujan was well aware that his proofs in prime number theory were not rigorous. In fact, later Ramanujan did send to Hardy some of his arguments in the theory of prime numbers, and indeed in his reply, Hardy carefully pointed out to Ramanujan that certain processes were not justified [7, pp. 87–89]. Similarly, Ramanujan's powerful procedure for evaluating large classes of integrals and summing many infinite series was lacking in rigor. Because Ramanujan was likely afraid to submit papers for publication wherein his results were not rigorously proven, requesting Narayana Aiyar to submit research announcements was a means by which he could claim some credit without subjecting his methodology to the demands of rigor.

Ramanujan's first two letters to Hardy were composed with the help of Narayana Aiyar, who strongly advised Ramanujan to accept Hardy's invitation to come to Cambridge. Narayana Aiyar and Ramanujan journeyed to the temple at Namakkal, where Goddess Namagiri gave Ramanujan permission to accept Hardy's bidding. Narayana Aiyar then had to arrange with Sir Francis Spring concessions in order that Ramanujan might take leave of his position with the Port Trust. Although Sir Francis Spring unflinchingly supported Ramanujan, he adhered to strict rules in the office. Narayana Aiyar often jokingly referred to him at home as "oosi milagai," a Tamil name for a variety of hot green chilies.

**4. MATHEMATICAL CONTRIBUTIONS.** In 1907, Narayana Aiyar became a founding member of the Indian Mathematical Club, which soon afterward became known as the Indian Mathematical Society. He served as the first assistant secretary from 1907 to 1910, and later served as treasurer from 1914 to 1928. Clearly, administrative duties in the positions he held after he left St. Joseph's College forestalled mathematical activity, but in 1907 he evidently gained new life, for he began to contribute problems and solutions to the *Educational Times*. Two years later, he commenced to submit and solve problems in the newly founded *Journal of the Indian Mathematical Club*, which in 1912 became the *Journal of the Indian Mathematical Society*. In addition to the two communications of Ramanujan's theorems to the Indian Mathematical Society, Narayana Aiyar himself published two further papers [5, 6] of his own in the *Journal of the Indian Mathematical Society* and an earlier one in the *Journal of the Indian Mathematical Club* [2].

Early in the twentieth century, mathematics taught in Indian schools and colleges reflected what was taught in England. The emphasis was on solving problems, instead of developing theories. The content of articles published in the *Journal of the Indian Mathematical Society* was not dissimilar in spirit to the questions and solutions published in the *Journal*. Moreover, if one examines Ramanujan's early papers and problems published in the *Journal of the Indian Mathematical Society*, several clearly reflect the nature of mathematics published in the same issues of the *Journal* and taught in India at that time. And Narayana Aiyar's problems and solutions also mirror the kind of mathematics popular in then-contemporary India. Narayana Aiyar's problems and solutions in the *Educational Times* will first be described. Then we briefly discuss his questions and solutions in the *Journal of the Indian Mathematical Club* and the *Journal of the Indian Mathematical Society*. The average difficulty of problems submitted to the *Educational Times* is perhaps higher than for problems submitted to the *Journal of the Indian Mathematical Club* and the *Journal of the Indian Mathematical Society*, and so we concentrate on the former set of problems.

During the years 1907–1911, S. Narayana Aiyar submitted a total of 26 problems to the *Educational Times*. During the same period, 14 of his solutions to problems posed by other contributors were also published in the *Educational Times*. In fact, except for one contributed solution, his proposed questions and solutions were all published during the years 1907–1911. Of the 26 posed problems, 12 are in analysis, mostly on integrals; 13 are in geometry, with most appearing in 1909 and 1910; and the remaining question was on determinants. Of the problems solved by Narayana Aiyar, only three were in analysis; four were in algebra, including determinants; seven were in geometry, toward the end of his period of solutions; one was in combinatorics; and one was in probability. Many of Narayana Aiyar’s problems, particularly in analysis and geometry, were extraordinarily difficult. In fact, for 13 of the 26 problems that he submitted, no solutions were published. Most of Narayana Aiyar’s problems in geometry are on elementary Euclidean geometry (mainly on conics), and were solved synthetically in the Apollonius style. The term “synthetic geometry” refers to Euclidian-type axiomatic geometry, in contrast to organizations of geometries based on coordinates or other analytic means. In personal communications, Illinois colleague John Wetzel opined, “I think it takes considerable ingenuity to use synthetic methods to solve such questions,” while Greek geometer A. Hatzipolakis remarked “they are interesting, and in my opinion deserve the attention of geometers.”

Those submitting and solving problems in the *Educational Times*, in addition to S. Narayana Aiyar, included several Indian mathematicians who also prominently contributed articles and problems to the *Journal of the Indian Mathematical Society*. These included M. T. Naraniengar, S. Narayanan, K. J. Sanjana, and V. Ramaswami Aiyar, the founder of the Indian Mathematical Society and the one whom Ramanujan approached in 1910 seeking a position in his office. Narayana Aiyar solved three problems submitted by Sanjana, who solved one of Narayana Aiyar’s problems. Recall that the second published paper by Ramanujan is entitled “On question 330 of Professor Sanjana” [12], [13, pp. 15–17]. “Question 330” refers to a problem posed by Sanjana in the problems section of the *Journal of the Indian Mathematical Society*. In [12], Ramanujan cleverly summed a class of infinite series, generalizing that in Sanjana’s problem. A problem posed by V. Ramaswami Aiyar was the one in elementary probability solved by S. Narayana Aiyar.

Although known everywhere by its abbreviated title, *The Educational Times*, the official name of this monthly periodical devoted to educational themes was the *Educational Times and Journal of the College of Preceptors*. *The Educational Times* was a well-respected periodical that was published in London and known throughout Europe and, of course, in India. It was first published in 1847 under the title *The Educational Times*, and it has continued up to the present day under several names with the current one being *The Educational Times: A Digest of Current Educational Literature*.

As indicated above, each issue contained a list of newly published problems as well as solutions to those previously published. At the end of the year, a separate volume entitled *Mathematical Questions and Solutions from the “Educational Times”* was published; herein we find the complete solutions that were published during the previous year. These volumes also contained occasional articles, and S. Narayana Aiyar published one such article in 1907 [1].

G. H. Hardy published 28 original problems and 16 solutions in *The Educational Times*. (Ironically, these numbers are almost identical with those of Narayana Aiyar.) The *first publication* of Hardy of any kind was Problem **13848**, which appeared on May 1, 1898, and which asked for the general equation, in symmetric form, of a circle through two fixed points. His solution appeared in the issue of August 1, 1898. (Note that the lag time between the publication of a problem and its solution

was considerably shorter in 1898 than it is with problem sections in contemporary journals.)

The first problem (**#11042**) that Narayana Aiyar solved was one that had been submitted in June, 1891, but evidently solutions were not received until late 1907 or early 1908 when three solutions, including that of Narayana Aiyar, were published in the February 1, 1908 issue. The problem is on a recurrence relation for Legendre polynomials. It is interesting to note that the second published solution was by G. N. Watson, who wrote over 30 papers on Ramanujan's work in the late 1920s and 1930s, and who, up until recent times, was more acquainted with Ramanujan's work than any other mathematician.

S. Narayana Aiyar's first question (**#16219**) was published in the issue of June 1, 1907 and is of historical interest. Narayana Aiyar had discovered that the solutions of certain differentials given by A. Cayley in his treatise *An Elementary Treatise on Elliptic Functions* were incorrect [10, Art. 414]. Narayana Aiyar's problem was to find the correct values. (Before we proceed further, it might be remarked that few "amateur" mathematicians (if any) would have been capable in those (or any) years of reading Cayley's *Elliptic Functions*.) Over the years, there has been extensive debate on the sources (if any) from which Ramanujan learned about elliptic functions before departing for England in 1914. And, if he had seen texts on the subject, was he aware of Cayley's treatise? There is considerable, although slightly conflicting, evidence that Ramanujan had been familiar with Cayley's *Elliptic Functions* in India [8], [9, pp. 9–15]. Narayana Aiyar's first published problem gives strong evidence that Ramanujan was acquainted with the book. Since Narayana Aiyar and Ramanujan were close friends, who worked on mathematics together during the evenings, it is inconceivable that Narayana Aiyar had not told Ramanujan about this book, if indeed Ramanujan had not already been aware of it before coming to Madras in 1910. Moreover, we also know that Ramanujan had made most of his voluminous contributions to elliptic functions, in particular to modular equations, in the two or three years prior to his departure for Cambridge, i.e., during the period when Ramanujan and Narayana Aiyar worked together.

We commented above on the interest of S. Narayana Aiyar's problems in geometry, and so we offer here two of them. Paul Yiu, in an article under preparation, plans to discuss all of Narayana Aiyar's geometrical problems. The first appeared in *The Educational Times* on June 1, 1910, but although four solutions were eventually published, they did not appear until 1912, indicating that it took a while for readers to work out solutions. The second appeared on August 1, 1910, and no solution was ever published.

**#16889** If a conic inscribed in a triangle passes through two points which are isogonal conjugates with respect to the triangle, then the tangents to this conic at these points intersect either in the incenter or in one of the excenters of the triangle.

**#16923** If the polar of the incenter (or an excenter) with respect to an inconic of a triangle passes through the circumcenter, then the pedal circle with respect to the triangle of the points in which the polar meets the conic touches the nine-point circle of the triangle.

As with Hardy, several of the initial problems that Narayana Aiyar submitted to *The Educational Times* asked for the evaluation of intriguing integrals. In his first problem (**#16238**) on integrals, published on July 1, 1907, readers were asked to evaluate

$$\int_0^\pi \frac{e^{mr \cos \theta}}{(a^2 - 2ar \cos \theta + r^2)^{\frac{1}{2}n}} \cos \left( mr \sin \theta \pm n \tan^{-1} \frac{r \sin \theta}{a - r \cos \theta} \right) d\theta.$$

Here,  $r < a$ , presumably  $n$  is a positive integer, and no conditions were prescribed for the parameter  $m$ . If one takes the plus sign above, Narayana Aiyar claims that the answer is simply  $\pi a^{-n}$ , but if the minus sign is taken, the evaluation is more complicated. There were no published solutions. Integrating a suitable function over the unit circle and using the residue theorem, M. D. Hirschhorn supplied the author with an elegant solution.

Trigonometric functions were featured in most of Narayana Aiyar's evaluations of integrals. We give one further example (slightly reformulated) (#16659), which is more difficult than the one above. Show that

$$\int_0^\infty \frac{\cos(\phi_1 + \phi_2 + \cdots + \phi_n) - \theta \sin(\phi_1 + \phi_2 + \cdots + \phi_n)}{\rho_1 \rho_2 \cdots \rho_n} \frac{d\theta}{1 + \theta^2} = \frac{1}{2}\pi,$$

where, for  $1 \leq j \leq n$ ,

$$\phi_j = \tan^{-1} \frac{jr \sin \theta}{1 - jr \cos \theta} \quad \text{and} \quad \rho_j = \sqrt{1 - 2jr \cos \theta + j^2 r^2}.$$

No solution was ever published, but Hirschhorn has solved this problem and generalized it; his generalized version appears as problem 11600 in this issue of the MONTHLY.

Lastly, we relate the elementary problem (#16187) on probability posed by V. Ramaswami Aiyar and solved by S. Narayana Aiyar. Tejashri and Atul play a series of games in which each game is won by Tejashri or Atul with equal probability, for a fixed contributed stake on each game. Atul has enough money to play an indefinite number of games, while Tejashri has no money but pretends that she also has sufficient funds to play indefinitely. Show that the probability that Tejashri will be able to maintain her pretension for  $2n - 1$  games or  $2n$  games is

$$\frac{1}{2} \frac{3}{4} \frac{5}{6} \cdots \frac{2n-1}{2n} = \frac{1}{2^{2n}} \binom{2n}{n}.$$

We now briefly turn to Narayana Aiyar's questions and solutions in the *Journal of the Indian Mathematical Society*. As previously indicated, the Indian Mathematical Society was originally called the Indian Mathematical Club, and the first three issues of the organization's journal, published in the years 1909–1911, were entitled the *Journal of the Indian Mathematical Club*. Prior to the publication of the *Journal of the Indian Mathematical Club*, problems were evidently published in *Progress Reports*. In the sixth, seventh, and eighth reports, Narayana Aiyar solved 4, 6, and 1 problems, respectively. In the first volume of the *Journal of the Indian Mathematical Club*, Narayana Aiyar published 11 problems and 15 solutions, with two of the solutions being those for his own problems. It is curious that in future issues there appear no further solutions by Narayana Aiyar to other submitted problems. Printed with the first issue are two *Progress Reports*, in which Narayana Aiyar proposed three additional problems, while in previous *Progress Reports*, he had proposed another three questions. The first proposed problem in the *Journal of the Indian Mathematical Club* is Question 59, indicating that the problems section was a continuation of those from the *Progress Reports*. Narayana Aiyar was one of seven mathematicians publishing a large number of problems and solutions in the maiden volume, with the others being S. Narayanan, N. B. Pendse, V. Ramaswami Aiyar, P. V. Seshu Aiyar, V. R. Sonti, and K. J. Sanjana. Recall that V. Ramaswami Aiyar founded the Indian Mathematical Club in 1907 and that P. V. Seshu Aiyar was Ramanujan's mathematics teacher at the Government College of Kumbakonam during the latter's one-year attendance at the

College. Clearly, all seven of these mathematical enthusiasts over a period of several years had built a storehouse of problems waiting to be published for the enjoyment and challenge of their contemporaries.

Almost all of the questions submitted by Narayana Aiyar in the initial issue of the *Journal of the Indian Mathematical Club* were, not surprisingly in view of our previous discussion, in geometry. In the next three years, he did not submit any new problems. He resumed the publication of problems in Volume 5 (published in 1913) of the *Journal of the Indian Mathematical Society*. Now his interest turned to series and integrals, which is not surprising, for he certainly came under the influence of Ramanujan, who became a clerk in S. Narayana's office in March, 1912. Ramanujan did not have a keen interest in geometry, as is clearly demonstrated in both his published papers [13] and notebooks [14]. However, Ramanujan loved integrals and series! Many of Narayana Aiyar's questions in 1915 and 1916 were on series of hypergeometric type, possibly reflecting Ramanujan's long-abiding interest in the subject, with Chapters 10 and 11 in his second notebook [14] devoted to the subject. Problems submitted by Narayana Aiyar in later years were generally in analysis and elliptic functions.

**Table 1.** Questions by S. Narayana Aiyar.

Volume and Year	No. of Proposed Questions
Progress Reports (1908)	6
1(1909)	11
2(1910)	0
3(1911)	0
4(1912)	0
5(1913)	2
6(1914)	4
7(1915)	7
8(1916)	2
9(1917)	1
10(1918)	1
15(1923–24)	3
16(1925–26)	2
17(1927–28)	2
18(1929–30)	1

**5. EPILOGUE.** Except for a few family pictures, only one photograph of S. Narayana Aiyar is extant. Narayana Aiyar was one of the delegates to the meeting of the Indian Mathematical Society in Bombay in 1919, and so can be seen in this group photo. This photograph is extremely valuable, because many important people in the life of Ramanujan are present. These include (numbering from the left) S. Narayana



Delegates to the Indian Mathematical Society meeting in Bombay, 1919. Narayana Aiyar is third from left in the first row.

Aiyar (3, first row), P. V. Seshu Aiyar (4, first row), V. Ramaswami Aiyar (5, second row), R. Ramachandra Rao (10, second row), and S. R. Ranganathan (2, third row). Ranganathan wrote the first book biography of Ramanujan in English [15]. He was the librarian at the University of Madras and is internationally recognized today as one of the founders of modern library science. Also mentioned in this article are M. T. Naraniengar (7, second row) and S. Narayanan (9, third row). For a recent account of the history of the Indian Mathematical Society and of mathematics in India during the past century, see the article by J. W. Dauben and R. Parikh [11].

We are grateful to V. Viswanathan for providing detailed information about his grandfather, S. Narayana Aiyar. Viswanathan's mother was Kamakshi, who entered the world in 1907 and possibly inspired her father to resume intense mathematical activity. Narayana Aiyar's son, N. Subbanarayanan [16], has written about his father and his father's relationship with Ramanujan. In particular, he writes that his father and Ramanujan exchanged slates prior to the latter's departure for England. In those days paper was expensive, and so, in particular, school children performed all of their work on slates. Before departing for England, Ramanujan did almost all of his mathematics on his slate. The bestowing of his slate to Narayana Aiyar clearly indicates the respect that Ramanujan had for him. Subbanarayanan tells that after Ramanujan's death he often went to the library to fetch books on elliptic functions for his father. Obviously, his love for the subject was not abated by the demise of Ramanujan.

Jim Tattersall supplied copies of all the problems and solutions that Narayana Aiyar submitted to *The Educational Times*, and also notified the author about the *Progress Reports*. Moreover, he informed us of [17], from which we took much of our information about *The Educational Times*. We are enormously beholden to Paul Yiu for providing solutions to several of Narayana Aiyar's problems in geometry, and to Mike Hirschhorn for solutions to some of Narayana Aiyar's difficult problems on integrals. We are also grateful to Antreas Hatzipolakis, Clark Kimberling, and John Wetzel for their comments. The referees also offered several helpful suggestions.

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